morning

D. 11/05/2015

Seat No.______

GANPAT UNIVERSITY

B.Tech. Sem.-II (ALL) CBCS REGULAR EXAMINATION, APRIL-JUNE 2015

SUBJECT: 2HS102 LINEAR ALGEBRA

Time: 3 hrs | Total marks: 60

Instruction: (1) All questions are compulsory.

(2) Write answer of each section in separate answer books.

(3) Figures to the right indicate marks of questions.

Section – I

Que-1 Answer the following

(a) Find the Eigen value and Eigen vector of the matrix $\begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}$ **4**

(b) Test for the consistency and solve the system $\begin{aligned} 2x+2y+z&=0 \\ x-y+3z&=0 \\ 2x+y-z&=0 \end{aligned}$ **3**

(c) Find the Rank of matrix $\begin{bmatrix} 0 & 1 & -3 & -1 \\ 1 & 0 & 1 & 1 \\ 3 & 1 & 0 & 2 \\ 1 & 1 & -2 & 0 \end{bmatrix}$ **3**

OR

Que-1 Answer the following.

(a) Find the Inverse of matrix $\begin{bmatrix} 1 & 2 & 5 \\ 2 & 3 & 1 \\ -1 & 1 & 1 \end{bmatrix}$ **4**

(b) Check whether the given vectors are L.D. or L.I.? If L.D. then find a relation between them $x_1=(1,-1,1)$, $x_2=(2,1,1)$, $x_3=(3,0,2)$ **3**

(c) Test for the consistency and solve the system $\begin{aligned} x+y+2z&=9 \\ 2x+4y-3z&=1 \\ 3x+6y-5z&=0 \end{aligned}$ **3**

Que-2 Answer the following.

(a) Verify Caley Hamilton theorem for the matrix $A=\begin{bmatrix} 1 & 2 & 4 \\ -1 & 0 & 3 \\ 3 & 1 & -2 \end{bmatrix}$ **4**

(b) Define : (1) Hermitian matrix (2) Skew- Hermitian matrix (3) Unitary Matrix **3**

(c) Prove that $\frac{1}{2}\begin{bmatrix} 1+i & -1+i \\ 1+i & 1-i \end{bmatrix}$ is an unitary matrix **3**

OR

Que-2 Answer the following.

(a) State the Cayley-Hamilton theorem and find A^{-1} for $A=\begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix}$ **4**

(b) If $A=\begin{bmatrix} 2+i & 3 & -1+3i \\ -5 & i & 4-2i \end{bmatrix}$, Show that $A^{*}A$ is a Hermitian matrix. **3**

(c) Express the matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & -1 & 4 \\ 3 & 1 & -1 \end{bmatrix}$ as a sum of symmetric and skew symmetric matrix 3

Que-3 (a) Diagonalise the matrix $A = \begin{bmatrix} -4 & -6 \\ 3 & 5 \end{bmatrix}$ 4

(b) Attempt any two 6

i Express the matrix $A = \begin{bmatrix} i & -i \\ 1+i & 2 \end{bmatrix}$ as a sum of Hermitian and Skew Hermitian matrix

ii Investigate for what values of λ & μ the equations $\begin{aligned} x+y+z &= 6 \\ x+2y+3z &= 10 \\ x+2y+\lambda z &= \mu \end{aligned}$

have (i) no solution (ii) a unique solution and (iii) an infinite no of solutions

iii If $A = \begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix}$ then using Caley Hemilton theorem prove that

$$A^5 - 3A^4 + A^3 - 7A^2 + 5A + I = 61I - 43A$$

Section – II

Que-4 **Answer the following.** 6

(a) Check wheather the set $M = \left\{ \begin{bmatrix} a & 1 \\ 1 & b \end{bmatrix} / a, b \in R \right\}$ is Vector space or not under the operations $\begin{bmatrix} a & 1 \\ 1 & b \end{bmatrix} + \begin{bmatrix} c & 1 \\ 1 & d \end{bmatrix} = \begin{bmatrix} a+c & 1 \\ 1 & b+d \end{bmatrix}$ and $\alpha \begin{bmatrix} a & 1 \\ 1 & b \end{bmatrix} = \begin{bmatrix} \alpha a & 1 \\ 1 & \alpha b \end{bmatrix}$ 4

(b) Show that the set $W = \{ a_0 + a_1x + a_2x^2 / a_0 + a_1 + a_2 = 0\}$ is a subspace under usual addition and scalar multiplication.

OR

Que-4 **Answer the following.** 6

(a) Show that $\mathbf{R}^3 = \{(x, y, z) / x, y, z \in R\}$ is a vector space under the usual operations 4

(b) Check whether the set $W = \{(x, y, z)/y = x + z + 1\}$ is a subspace or not under usual vector addition and scalar multiplication.

Que-5 **Answer the following.** 4

(a) Find Range and Kernel of linear Transformations of $T: R^3 \to R^2, T(x, y, z) = (x + y, y + z)$ also verify the Rank – Nullity Theorem. 3

(b) Check whethet the function $T: R^2 \to R^2, T(x, y) = (x + 1, y)$ are L. T. or not ? 3

(c) Test the convergence of $\frac{1}{1 \cdot 2} + \frac{1}{2 \cdot 3} + \frac{1}{3 \cdot 4} + \cdots \infty$

OR

Que-5 **Answer the following.** 4

(a) Define range and kernal also find rang and kernal of linear transformation $T: R^3 \to R^2, T(x, y, z) = (x + y, y)$. 3

(b) Determine if the vectors $v_1 = (1,3,1), v_2 = (2,-1,1)$ and $v_3 = (1,1,4)$ will be span R^3 ? 3

(c) Test the convergence of $\sum_{n=2}^{\infty} \frac{1}{n \log n}$. 4

Que-6 (a) Express the polynomial $P = -9 - 7x - 15x^2$ as a linear combination of $P_1 = 2 + x + 4x^2$, $P_2 = 1 - x + 3x^2$, $P_3 = 3 + 2x + 5x^2$. 6

(b) **Attempt any two.**

i Show that $v_1 = (1,-1,1)$, $v_2 = (0,1,2)$, $v_3 = (3,0,-1)$ forms a basis for R^3.

ii Check wheather the alternating series $\sum_{n=1}^{\infty} (-1)^n \frac{n+3}{n^2+2}$ is convergent or not ?

iii Give one example of set which is not a Vector space also justify your answer.

END OF PAPER