3EC101 **Seat No.**

GANPAT UNIVERSITY

M. Tech. Semester I (EC) Examination, November/December 2012

Communication Mathematics

Max. Time: 3 Hrs. **[Max. Marks: 70**

Instructions:

1. Attempt **all** questions.
2. Answers to the two sections must be written in separate answer books.
3. Figures to the **right** indicate full marks.
4. Assume suitable data, if necessary.
5. Question numbers three and six are compulsory.

SECTION-I

1 (A) A bag contains 5 balls and it is not known how many of them are white. Two balls are drawn at random from the bag and they are noted to be white. What is the chance that all the balls in the bag are white? 4

(B) Given $f_{xy}(x,y) = cx(x-y), 0 < x < 2, -x < y < x,$ and 0 elsewhere. (i)Evaluate c, (ii) find $f_x(x)$, (iii) $f_{y/x}(y|x)$ and (iv) $f_y(y)$. 6

(C) For two independent events A and B, prove that two events $\bar{A}$ and B are also independent. 2

OR

1 (A) An urn contains 10 white and 3 black balls. Another urn contains 3 white and 5 blackballs. Two balls are drawn at random from the first urn and placed in the second urn and then 1 ball is taken at random from the latter. What is the probability that it is a white ball? 4

(B) If the random variable X takes the values 1, 2, 3 and 4 such that $2P(X=1) = 3P(X=2) = P(X=3) = 5P(X=4)$, find the probability density function and cumulative distribution function. 6

(C) If random variable X is exponentially distributed with parameter λ=1, find the moment generating function of X. 2

2 (A) If X and Y are independent random variables with identical distribution defined as $U(0,1)$, show that the pdf of Z is the convolution of pdfs of X and Y, where Z=X+Y. 5

(B) Write short note on Output statistics of Linear Systems. 6

OR

2 (A) Given the uncorrelated random variables X_1, X_2 and X_3, whose means are 2, 1 and 4 and whose variance are 9, 20 and 12, find (i) mean and variance of X_1-$2X_2$+$5X_3$, (ii) the covariance between X_1+$5X_2$ and $2X_2$-X_3+5. 5

(B) Write short note on Stationary Random Process. 6

3 **(A)** If X and Y are independent RVs with pfd's $e^{-x}, x \geq 0$ and $e^{-y}, y \geq 0$ respectively, find the density function of $U = \frac{X}{X+Y}$ and $V = X + Y$. Are U and V independent? **6**

(B) Show that the random process X(t)=A Cos(ωt + θ), where θ is uniformly distributed random variable in the range of (0, 2π) and A, ω and t are constants, is weakly stationary random process. **4**

(C) State and prove the Tchebychev's Inequality. **2**

SECTION II

4 **(A)** For which rational value of a does the following system have (i) no solutions (ii) exactly one solution (iii) infinitely many solutions? **6**

$$\begin{aligned} x + 2y - 3z &= 4 \\ 3x - y + 5z &= 2 \\ 4x + y + (a^2 - 14)z &= a + 2 \end{aligned}$$

(B) Determine explicitly the inverse of the following 3 × 3 elementary row matrices: **6**
(i)$E_{12}E_{23}$ (ii) $E_{12}(3)E_{21}(-3)$

OR

4 **(A)** Solve the following systems of linear equations by reducing the augmented matrix to reduced row-echelon form: **6**

(i) $$\begin{aligned} x + y + z &= 2 \\ 2x + 3y - z &= 8 \\ x - y - z &= -8 \end{aligned}$$

(ii) $$\begin{aligned} 3x - y + 7z &= 0 \\ 2x - y + 4z &= 1/2 \\ x - y + z &= 1 \\ 6x - 4y + 10z &= 3 \end{aligned}$$

(B) Let $A = \begin{bmatrix} 2 & 2 & 4 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$. Prove that A is non-singular, find A^{-1} and express A as a product of elementary row matrices. **6**

5 **(A)** If $A = \begin{bmatrix} 4 & -3 \\ 1 & 0 \end{bmatrix}$, use the fact that $A^2 = 4A - 3I_2$ and mathematical induction to prove that $A^n = \frac{(3^n - 1)}{2}A + \frac{(3 - 3^n)}{2}I_2$ if n ≥ 2. **6**

(B) Find the basis for the row space and column space of the matrix **5**

$$A = \begin{bmatrix} 1 & 3 & -5 & 1 & 5 \\ 1 & 4 & -7 & 3 & -2 \\ 1 & 5 & -9 & 5 & -9 \\ 0 & 3 & -6 & 2 & -1 \end{bmatrix}.$$

OR

5 **(A)** Explain how to use the LU factors of a nonsingular matrix $A_{n \times n}$ to compute A^{-1} efficiently. **4**

(B) Find the eigenvalues and eigen vectors of the matrix **7**

$$A = \begin{bmatrix} 1 & -6 & -4 \\ 0 & 4 & 2 \\ 0 & -6 & -3 \end{bmatrix}.$$

6 (A) Find the least square solution of equation AX=Y, where 7

$$A = \begin{bmatrix} 2 & 1 \\ 1 & 0 \\ 0 & -1 \\ -1 & 1 \end{bmatrix}, \quad Y = \begin{bmatrix} 3 \\ 1 \\ 2 \\ -1 \end{bmatrix}$$

(B) Let $A = \begin{bmatrix} 1 & 4 & 5 \\ 4 & 18 & 26 \\ 3 & 16 & 30 \end{bmatrix}$, determine the lower triangular matrix L of A. 5

END OF PAPER