morning Date: 30/12/13.

Seat No. ________________

GANPAT UNIVERSITY

M. Tech. Semester I (EC) Regular Examination, December 2013

3 EC 101: Communication Mathematics

Max. Time: 3 Hrs.] [Max. Marks: 70

Instructions:

1. Attempt **all** questions.
2. Answers to the two sections must be written in separate answer books.
3. Figures to the **right** indicate full marks.
4. Assume suitable data, if necessary.
5. Question numbers three and six are compulsory.

SECTION-I

1 (A) A and B alternatively throw a pair of dice. A wins if he throws 4 before B throws 5 and B wins if he throws 5 before A throws 4. If A begins, find the probability of A winning. 4

(B) The joint pdf of the RV (X, Y) is given by $f(x,y) = kxye^{-(x^2+y^2)}$, $x > 0, y > 0$. Find the value of k and prove also that X and Y are independent. 6

(C) If A and B are independent events, prove that $P(A \cup B) = 1 - P(\bar{A})P(\bar{B})$. 2

OR

1 (A) An urn contains 5 white balls, 3 red balls and 4 black balls. Two are drawn from the urn at random. Find the probability that (i) both of them are of same colour and (ii) they are of different colours. 4

(B) The joint probability density function of (X, Y) is given by $p(x,y) = k(2x + 3y)$, $x = 0,1,2; y = 1,2,3$. Find all the value of k and all the marginal probabilities. 6

(C) State and prove the Tchebychev's Inequality. 2

2 (A) If X and Y are independent RVs with identical uniform distributions in (0, 1), find the joint pdf of (U, V), where U = X+Y and V=X-Y. 4

(B) Describe the statistical response of linear systems to the random input. 4

(C) Suppose that the random variable X satisfies $E[X]=0$, $E[X^2]=1$, $E[X^3]=0$ and $E[X^4]=3$ and let $Y = a + bX + cX^2$. Find the correlation coefficient $\rho(X, Y)$. 3

OR

2 (A) If the joint pdf of (X, Y) is given by $f(x,y) = x + y; 0 \le x, y \le 1$, find the pdf of $U = XY$. 4

(B) Write short note on White Noise Random Process. 5

(C) Find the variance of a Gaussian random variable. 2

3 (A) If X and Y are independent RVs with pfd's $e^{-x}, x \ge 0$ and $e^{-y}, y \ge 0$ respectively, find the density function of $U = \frac{X}{X+Y}$ and $V = X + Y$. Are U and V independent? 8

(B) If random variable X is exponentially distributed with parameter λ=1, find the moment generating function of X. 4

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SECTION-II

4 (A) For the following system find in which case system has consistent solution? Also find the solution. 8

$$\begin{aligned} 2x - y + 3z &= a \\ 3x + y - 5z &= b \\ -5x - 5y + 21z &= c \end{aligned}$$

(B) Determine explicitly the inverse of the following 3×3 elementary row matrices: $E_{12}E_{21}(-3)$ 4

OR

4 (A) Using LU factorization solve the following system. 8

$$\begin{aligned} x + y + z &= 2 \\ 2x + 3y - z &= 8 \\ x - y - z &= -8 \end{aligned}$$

(B) Let $A = \begin{bmatrix} 2 & 2 & 4 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$. Prove that A is non-singular, find A^{-1}.

5 (A) If $A = \begin{bmatrix} 7 & 4 \\ -9 & -5 \end{bmatrix}$. Prove that $A^n = \begin{bmatrix} 1+6n & 4n \\ -9n & 1-6n \end{bmatrix}$ if $n \geq 1$. 5

(B) Determine the basis for the nullspace of the following matrix. 6

$$A = \begin{bmatrix} -1 & 2 & -1 & 5 & 6 \\ 4 & -4 & -4 & -12 & -8 \\ 2 & 0 & -6 & -2 & 4 \\ -3 & 1 & 7 & -2 & 12 \end{bmatrix}.$$

OR

5 (A) Let $A = \begin{bmatrix} 1 & 1 & -1 \\ 0 & 0 & 1 \\ 2 & 1 & 2 \end{bmatrix}$, use the fact that $A^3 = 3A^2 - 3A + I_3$ to express A^4 in terms of A^2, A and I_3 and hence calculate A^4. 5

(B) Find the eigenvalues and eigen vectors of the matrix 6

$$A = \begin{bmatrix} 4 & 2 & -2 \\ -5 & 3 & 2 \\ -2 & 4 & 1 \end{bmatrix}.$$

6 (A) Find the least square solution of the equation AX=Y, where 8

$$A = \begin{bmatrix} 2 & 1 \\ 1 & 0 \\ 0 & -1 \\ -1 & 1 \end{bmatrix}, \quad Y = \begin{bmatrix} 3 \\ 1 \\ 2 \\ -1 \end{bmatrix}$$

(B) Briefly explain Singular Value Decomposition. 4

END OF PAPER

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