Exam No. _____

GANPAT UNIVERSITY

M. Tech. Semester I (EC) Regular Examination, NOV-DEC 2014
3 EC 101: Essential Communication Mathematics

Max. Time: 3 Hrs.] [Max. Marks: 60

Instructions:

1. Attempt **all** questions.
2. Answers to the two sections must be written in separate answer books.
3. Figures to the **right** indicate full marks.
4. Assume suitable data, if necessary.
5. Be precise and to the point in answering the descriptive questions.

SECTION-I

1 (A) A class consisting of 4 graduate and 12 undergraduate students is randomly divided into 4 groups of 4. What is the probability that each group includes a graduate student? 5

(B) Verify whether following is a probability density function or not. If yes, find the distribution function. 5

$$f(x) = |x|, |x| < 1$$
$$= 0, otherwise$$

OR

1 (A) A bag contains 5 balls and it is not known how many of them are white. Two balls are drawn at random from the bag and they are noted to be white. What is the chance that all the balls in the bag are white? 4

(B) Find the density function of Y = aX + b in terms of the density function of random variable X. Let X be a continuous random variable with pdf 6

$$f(x) = \begin{cases} \frac{x}{12}, in\ 1 < x < 5 \\ 0, otherwise \end{cases}$$

Find the probability density function of Y = 2X -3.

2 (A) If X and Y are independent uniform random variables in the common interval (0, 1), show that the pdf of Z is the convolution of pdfs of X and Y, where Z=X+Y. 6

(B) Derive the equations of Output statistics of Linear Systems. 4

OR

2 (A) If the joint pdf of (X, Y) is given by $f(x, y) = x + y; 0 \le x, y \le 1$, find the pdf of $U = XY$. 6

(B) Write short note on White Noise Random Process. 4

3 (A) What is a Moment Generating Function (MGM)? Find the MGM of exponential random variable. 6

(B) Prove the Markov and Tchebychev inequalities. 4

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SECTION-II

4 (A) For the following system find the conditions in which (i) No solution, (ii) Unique solution and (iii) infinitely many solutions. 6

$$\begin{aligned} x - 2y + 3z &= 4 \\ 2x - 3y + az &= 5 \\ 3x - 4y + 5z &= b \end{aligned}$$

(B) Let A be the following product of 4 X 4 elementary row matrices: $A = E_3(2)E_{14}E_{12}(3)$. Find A and A^{-1} explicitly. 4

OR

4 (A) Using LU factorization solve the following system. 6

$$\begin{aligned} -2x + y - 3z &= 4 \\ x + 2y + 3z &= 13 \\ 3x + z &= -1 \end{aligned}$$

(B) Let $A = \begin{bmatrix} 1 & 1 & 1 \\ -1 & 1 & 0 \\ 2 & 0 & 0 \end{bmatrix}$. Prove that A is non-singular, find A^{-1} using Gauss Jordan method. 4

5 (A) If $A = \begin{bmatrix} 2a & -a^2 \\ 1 & 0 \end{bmatrix}$. Prove that $A^n = \begin{bmatrix} (n+1)a^n & -na^{n+1} \\ na^{n-1} & (1-n)a^n \end{bmatrix}$ if $n \geq 1$. 5

(B) Find the basis for the space spanned by the vectors 5
$v_1 = (1, -2, 0, 0, 3)$, $v_2 = (2, -5, -3, -2, 6)$, $v_3 = (0, 5, 15, 10, 0)$, $v_4 = (2, 6, 18, 8, 6)$.

OR

5 (A) Let $A = \begin{bmatrix} 1 & 1 & -1 \\ 0 & 0 & 1 \\ 2 & 1 & 2 \end{bmatrix}$, use the fact that $A^3 = 3A^2 - 3A + I_3$ to express A^4 in terms of A^2, A and I_3 and hence calculate A^4. 4

(B) Find the eigenvalues and eigen vectors of the matrix 6

$$A = \begin{bmatrix} 3 & 10 & 5 \\ -2 & -3 & -4 \\ 3 & 5 & 7 \end{bmatrix}.$$

6 (A) Find the least square solution of equation $AX = Y$, where 6

$$A = \begin{bmatrix} 2 & 1 \\ 1 & 0 \\ 0 & -1 \\ -1 & 1 \end{bmatrix}, \quad Y = \begin{bmatrix} 3 \\ 1 \\ 2 \\ -1 \end{bmatrix}$$

(B) Write short note on Singular Value Decomposition. 4

END OF PAPER