Exam No. ____________

GANPAT UNIVERSITY

M. Tech. Semester I (EC) Regular Examination NOV-DEC 2015

3 EC 101: ESSENTIAL COMMUNICATION MATHEMATICS

Max. Time: 3 Hrs.] **[Max. Marks: 60**

Instructions:

1. This Question paper has two sections. Attempt each section in separate answer book.
2. Figures on right indicate marks.
3. Assume suitable data, if necessary.
4. Be precise and to the point in answering the descriptive questions.

SECTION-I

1 **(A)** For four independent events A_1, A_2, A_3, A_4, if $P(A_3 \cap A_4) > 0$ show that 4

$P(A_1 \cup A_2 \mid A_3 \cap A_4) = P(A_1 \cup A_2)$.

(B) The joint probability density function (pdf) of random variables X and Y is given by 6

$$f_{X,Y}(x,y) = k(2x+3y), \quad x = 0,1,2; \quad y = 1,2,3.$$

Find the value of k and all the marginal probabilities.

OR

1 **(A)** An urn contains 10 white and 3 black balls. Another urn contains 3 white and 5 blackballs. Two balls are drawn at random from the first urn and placed in the second urn and then 1 ball is taken at random from the latter. What is the probability that it is a white ball? 4

(B) State the pdf of following distributions and find their mean and variance. 6

i) Uniform; ii) Poisson.

2 **(A)** What is a Moment Generating Function (MGM)? Find the MGM of exponential random variable. 5

(B) Briefly explain the stationarity of Random Process. 5

OR

2 **(A)** Given the uncorrelated random variables X_1, X_2 and X_3, whose means are 2, 1 and 4 and whose variance are 9, 20 and 12, find (i) mean and variance of $X_1-2X_2+5X_3$, (ii) the covariance between X_1+5X_2 and $2X_2-X_3+5$. 6

(B) State and prove the Tchebychev and Markov inequalities. 4

3 **(A)** For two independent random variables X and Y having identical uniform distributions in (0, 1), find (i) the joint pdf of (U, V), where U = X+Y and V=X-Y, (ii)Also find the marginal pdf of U and V. 6

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(B) Show that the random process $X(t) = A\cos(\omega t + \theta)$, where θ is uniformly distributed random variable in the range of $(0, 2\pi)$ and A and ω, t are constants, is weakly stationary random process. 4

SECTION II

4 (A) For which rational value of a does the following system have (i) no solutions (ii) exactly one solution (iii) infinitely many solutions? 6

$$x + 2y - 3z = 4$$
$$3x - y + 5z = 2$$
$$4x + y + (a^2 - 14)z = a + 2$$

(B) Determine explicitly the following products of 3 × 3 elementary row matrices: 4

(i)$E_{12}E_{23}$ (ii) $E_{12}(3)$ (iii) E_{12}^{-1} (iv) $E_3(-2)$.

OR

4 (A) Solve the following systems of linear equations by reducing the augmented matrix to reduced row-echelon form: 6

(i) $\begin{aligned} x + y + z &= 2 \\ 2x + 3y - z &= 8 \\ x - y - z &= -8 \end{aligned}$ (ii) $\begin{aligned} 3x - y + 7z &= 0 \\ 2x - y + 4z &= 1/2 \\ x - y + z &= 1 \\ 6x - 4y + 10z &= 3 \end{aligned}$

(B) Let $A = \begin{bmatrix} 1 & 2 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$. Prove that A is non-singular, find A^{-1}. 4

5 (A) Prove the following: 5

(i)If A is $m \times n$ and B is $n \times m$ such that $AB = I_m$ and $BA = I_n$, the $m = n$.

(ii) If A, B and A + B are each nonsingular, prove that

$$A(A+B)^{-1}B = B(A+B)^{-1}A = (A^{-1} + B^{-1})^{-1}$$

(B) Find the basis for the row space and column space of the matrix 5

$$A = \begin{bmatrix} 1 & 3 & -5 & 1 & 5 \\ 1 & 4 & -7 & 3 & -2 \\ 1 & 5 & -9 & 5 & -9 \\ 0 & 3 & -6 & 2 & -1 \end{bmatrix}.$$

OR

5 Find the eigenvalues and eigen vectors of the matrix 10

$$A = \begin{bmatrix} 2 & -2 & 5 \\ 1 & -6 & 4 \\ 1 & 3 & 2 \end{bmatrix}.$$

6 (A) Find the least square solution of equation Ax=b, where 7

$$A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 1 \end{bmatrix}, \quad b = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}.$$

(B) Explain how to use the LU factors of a nonsingular matrix $A_{n\times n}$ to compute A^{-1} efficiently. 3

END OF PAPER