Seat No___________

GANPAT UNIVERSITY
M.TECH SEM.-II INFORMATION TECHNOLOGY
REGULAR JULY 2013 EXAMINATION
3IT205: ESSENTIAL MATHEMATICS

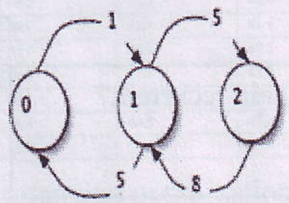
Max Time : 3 Hour] [Total Marks : 70

Instructions:
1. All questions are compulsory
2. Figures to the right indicate full marks.
3. Answer Both Sections in Separate Answer sheets

SECTION-I

Q-1 Answer the following:

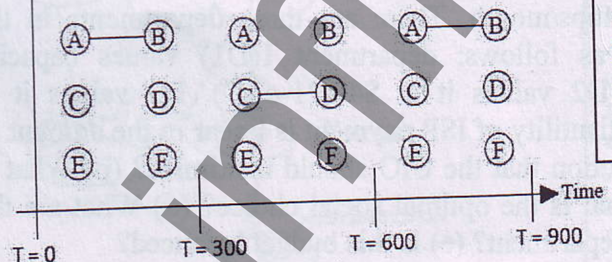
[A] [4]



Find the equilibrium probabilities of being in each state for the birth-death process.

[B] For M/M/1 queue, consider a link to which packets arrive as a Poisson process at a rate of 200 packets/sec such that the time taken to service a packet is exponentially distributed. Suppose that the mean packet length is 400 bytes, and that the link capacity is 1.0 Mbps. What is the probability that the link's queue has 1, 2 and 10 packets respectively? [4]

[C] [4]



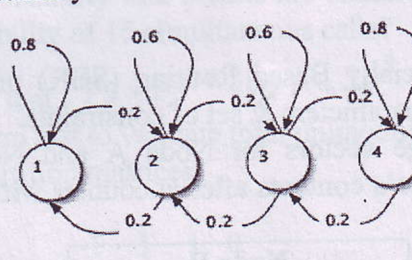
Compute temporal distance for all nodes and show steps of matrix building.

OR

Q-1 Answer the following:

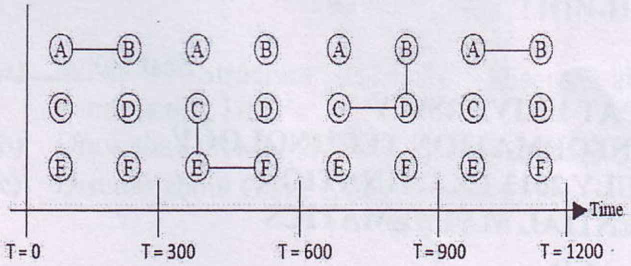
[A] Consider the same system as in question above Q.1(b) with but with the restriction that the queue only has four buffers. What is the probability that three of these are in use? How many buffers should we provision to ensure that the blocking probability is no more than 10-6? [4]

[B] [4]



For given Markov chain:
1.Compute Stationary probability 2. Compute Residence Time for each state of markov chain.

[C]



Compute temporal distance for all nodes and show steps of matrix building. [4]

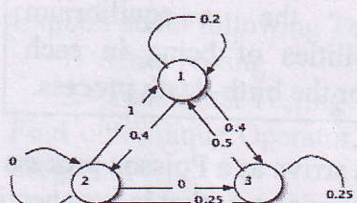
Q-2 Answer the following

[A] Consider the following payoff matrix for a two-player zero-sum game: [6]

	C1	C2
R1	(3,-3)	(1,-1)
R2	(2,-2)	(4,-4)

Compute the maximum strategy for both players.

[B] [5]



Is state 1 in the chain recurrent? Compute f_1^1, f_1^2, f_1^3

OR

Q-2

[A] The CIO of a company wants to decide how much capacity to buy from its ISP. The cost of capacity is \$20/ Mbps/month. There are three departments in the company, who value capacity as follows: department 1(D1) values capacity xMbps/month at \$20(*1-$e^{-0.5x}$), D2 values it at \$40(*1-$e^{-0.5x}$), D3 values it at \$80(*1-$e^{-0.5x}$). (a) Assuming the disutility of ISP payment is linear in the amount of payment, what is the overall function that the CIO should maximize? (b) What is type of each department? (c) What is the optimal social choice? (d) What are the Clarke Pivot payments for each department? (e) Is this budget balanced? [6]

[B] Consider that , a person is on an infinite staircase on stair number 10 at time 0 and potentially moves once every clock tick. Suppose that he moves from stair *i* to stair *i+1* with probability 0.2, and from stair *i* to stair *i-1* with probability 0.2 (the probability of staying on stair *I* is 0.6). [5]

Compute the probability that the person is on each stair at time 1 (after the first move), time 2, and time 3.

Q-3 Answer the following :

[A] Show the mathematical formulation of Socially Based Routing (SBR) protocol with objective function, input, output, fixed parameters & set of constraints. [6]

[B] Following table shows the current message vectors for Node A and Node B respectively. Show respective message vector's contents after encounter with each other for epidemic routing. [6]

Node A	
Dest Id	Seq.No
D	1
G	1

Node B	
Dest ID	Seq No
D	0
E	0

SECCTION -II

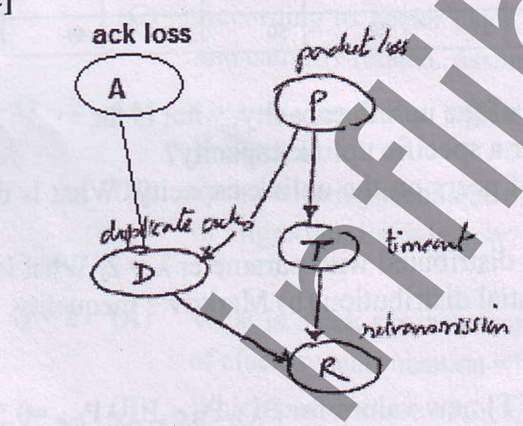
Q-4

[A] A university is connected to the Internet using three ISPs. To test their relative performance, the IT staff conduct an experiment where they measured the ping times to a well-known website over each of the three providers over a period of ten days. The mean ping time using each ISP on each day is shown below. Use single-factor ANOVA to test the hypothesis that the ISPs are statistically identical. **[4]**

Day	ISP1	ISP2	ISP3
1	41.2	50.7	41.1
2	34.9	38.5	48.2
3	43.5	56.3	73.2
4	64.2	54.2	48.4
5	64.0	46.4	61.4
6	54.9	58.4	43.2
7	59.3	61.8	63.9
8	73.1	69.4	54.3
9	56.4	66.3	67.4
10	63.8	57.4	58.4

[B] Use Lagrangian optimization to find the optimal value of $z=x^3+4y$ subject to the condition that $x^2+y^2=1$ (i.e. the points (x,y) lie on the unit circle. **[4]**

[C] **[4]**



Consider the Bayesian network in Figure. It shows that when there is a packet loss (cause) there can be a timeout at TCP transmitter (effect). Similarly, on the loss of an acknowledgement, there can also be a timeout. A packet loss can lead to a duplicate acknowledgment being received at the transmitter. Packet and ack loss are mutually exclusive, as are duplicate acks and timeouts. And, finally, if there is either a duplicate ack or a timeout at the transmitter, it will retransmit a packet.

Find P(ack loss | retransmission)

OR

Q-4

[A] A hotel has 20 guest rooms. Assuming outgoing calls are independent and that a guest room makes 10 minutes worth of outgoing calls during the busiest hour of the day, what is the probability that 5 calls are simultaneously active during the busiest hour? What is the probability of 15 simultaneous calls? **[4]**

[B] Prove that : $\mu_3 = \mu_3' - 3\mu_2'\mu + 2\mu^3$ **[4]**

[C] Perform test to validate total unimodularity (TU) of given matrices. If it is TU then derive the partition matrices. **[4]**

$$A = \begin{matrix} -1 & 1 & -1 & 0 \\ -1 & 0 & 0 & -1 \\ 0 & -1 & 0 & 1 \\ 1 & 0 & -1 & 0 \end{matrix}$$

Q-5 Answer the following questions.

[A] Suppose you have K balls that need to placed in M urns such that the payoff from placing the kth ball in the mth urn is pkm, and no more than 2 balls can be placed in each urn. Model this as a weighted bipartite matching problem. **[4]**

[B] Prove that : $X \sim N(\mu,\sigma^2)$ then $(X-\mu)/\sigma \sim N(0,1)$. **[3]**

[C] Consider the following probability mass function defined jointly over the random variables, X, Y, and Z: **[4]**
P(000) = 0.05; P(001) = 0.05; P(010) = 0.1; P(011)=0.3;P(100) = 0.05; P(101) = 0.05; P(110) = 0.1; P(111)=0.3.
(a) Write *down* $pX, pY, pZ, pXY, pXZ, pYZ$.
(b) Are X and Y, X and Z, or Y and Z independent? What is the probability that $X=0$ given that $Z=1$?

OR

Q-5

[A] Suppose you know that the objective function you are trying to maximize has no more than K local optima. Outline an algorithm that is guaranteed to find the global optimum using hill climbing. **[4]**

[B] A researcher measures the mean uplink bandwidth of 10 desktop computers (in kbps) as well their mean number of peer-to-peer connections over the period of one hour, obtaining the following data set: **[3]**

Uplink capacity	202	145	194	254	173	94	102	232	183	198
# peers	50	31	47	50	41	21	24	50	41	49

a) If the number of peers were independent of the uplink capacity, what is the expected value of the number of peers for a specific uplink capacity?
b) Compute the regression of the number of peers on the uplink capacity. What is the slope of the best-fit line.

[C] Consider a random variable X that exponentially distributed with parameter $\lambda = 2$. What is the probability that $X > 10$ using (a) the exponential distribution (b) Markov's inequality. **[4]**

Q-6 Answer the following question.

[A] Compute the **delivery predictability [PROPHET]** new values for $P_{A,B}$ $P_{B,C}$,$P_{A,C}$ P_{init} =0.55 ,β =0.45 **[6]**

From/To	B	C
A	0.4	0.6
B	0.8	0.2

[B] Use Bayesian network framework for DTN routing and discuss Forwarding and Classification phase. **[6]**

-------- X-------- X--------X