Seat No___________

GANPAT UNIVERSITY
M.TECH SEM.-II INFORMATION TECHNOLOGY
MAY-JUNE 2014 EXAMINATION
3IT205: ESSENTIAL MATHEMATICS

Max Time : 3 Hours] **[Total Marks : 70**

Instructions:
1. All questions are compulsory
2. Figures to the right indicate full marks.
3. Answer both sections in separate answer sheets

SECTION-I

Q-1

[A] Use Lagrangian optimization to find the optimal value of $z=x^3+2y$ subject to the condition that $x^2+y^2=1$ (i.e. the points (x,y) lie on the unit circle). [6]

[B] Consider a company that has two network connections to the Internet through two providers (this is also called *multi-homing*). Suppose that the providers charge per-byte and provide different delays. For example, the lower-priced provider may guarantee that transit delays are under 50ms, and the higher-priced provider may guarantee a bound of 20ms. Suppose the company has two commonly used applications, A and B that have different sensitivities to delay. Application A is more tolerant of delay than application B. Moreover, the applications, on average, generate a certain amount of traffic every day, all of which has to be carried by one of the two links. The company wants to allocate *all* the traffic from the two applications on the two links, maximizing their benefit while minimizing its payments to the link providers. [6]

Represent the problem in standard form.

OR

Q-1

[A] Model the network flow problem where the warehouses have infinite bounded capacity as a linear program. [6]

[B] Two users, Alice and Bob, can schedule jobs on a machine in one of two time periods, Period 1 or Period 2. If Alice schedules a job during Period 1, she gains a benefit of 20 units, and incurs a cost of 10 units, and during Period 2, she gains a benefit of 10 units and incurs a cost of 20 units. If Bob schedules a job during Period 1, he gains a benefit of 100 units and incurs a cost of 10 units, and during Period 2, he gains a benefit of 10 units and incurs a cost of 200 units. Each user may schedule at most one job in one time unit and in each time period, at most one job can be scheduled. [6]
Generalize this to the case where n users can schedule jobs on one of k machines, such that each user incurs a specific cost and gains a specific benefit on each machine at each of m time periods. Write out the ILP for this problem.

Q-2

[A] Perform test to validate total unimodularity (TU) of given matrices. If it is TU then derive the partition matrices. [6]

$$A = \begin{matrix} 1 & -1 & -1 & 0 \\ -1 & 0 & 0 & 1 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 0 \end{matrix}$$

[B] Suppose you have K balls that need to placed in M urns such that the payoff from placing the kth ball in the mth urn is pkm, and no more than 2 balls can be placed in each urn. Model this as a weighted bipartite matching problem. [5]

OR

Q-2

[A] Suppose you know that the objective function you are trying to maximize has no more than K local optima. Outline an algorithm that is guaranteed to find the global optimum using hill climbing. [6]

[B] Geometrically find the optimal value of O where [5]
$O = 6x_1 + x_2 - 2x_3$ and $x_1 + x_2 + x_3 = 1$ where, $x_1 \geq 0\ x_2 \geq 0\ x_3 \geq 0$

Q-3 A pair of standard, balanced six-sided dice is rolled. [12]
(i) What is the probability that the number rolled is 12?
(ii) What is the probability that the number rolled is 7?
(iii) If it is known to us that the number rolled is odd, what is the probability that the number rolled is 7?

SECTION-II

Q-4

[A] X is a discrete random variable. Write neatly and clearly the formulas for expected value E(X) and variance Var(X) of X. **[4]**

[B] Discrete random variable X has values 10, 20, 30, 40, 50 and 60. The probabilities of these values are, respectively: 0.1, 0.1, 0.15, 0.15, 0.25 and 0.25. Random variable Y is defined as 4*X. Find E(Y) and Var(Y). **[8]**

OR

Q-4 Continuous random variable X has probability density function: **[12]**

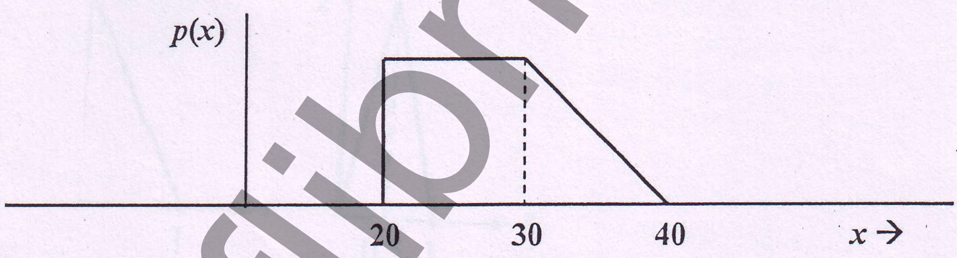
$p(x) = k*[1-(x-5)^2]$, for $4 <= x <= 6$; and $p(x) = 0$ outside this range.

Find the value of k, and find the probability Pr (4.2 <= X <= 5.8).

Q-5 An amplifier is produced with power gain G which follows a normal (Gaussian) distribution with mean $\mu = 2000$ and standard deviation $\sigma = 40$. A profit of of Rs. 2000/- is made on each amplifier sold which satisfies G >= 1900. What is the approximate expected profit if a total of 200 amplifiers are sold? **[11]**

OR

Q-5 A continuous random variable X has the following probability density function. Find the expected value and standard deviation of X. **[11]**



Q-6 H is a Boolean random variable which represents HEAVY RAIN, and D is a Boolean random variable which represents DISEASE in a city. D is dependent on H, as seen in the following table of conditional probability: **[12]**

	H	¬H
D	0.9	0.2
¬D	0.1	0.8

The unconditional probability of H is Pr(H) = 0.1 and Pr(¬H) = 0.9. Using Bayes' rule, find the conditional probability (H | ¬D).

END OF PAPER